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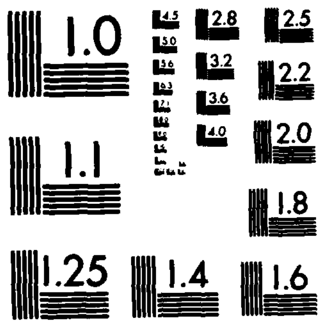

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A NEW ADAPTIVE ANTENNA SYSTEM FOR COHERENT SIGNALS AND INTERFERENCE

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ABSTRACT

In this paper we introduce a new adaptive antenna system able to work well even when the desired signal and the interference are coherent. The present adaptive beamformers fail to operate in these cases. The results of simulations appear to confirm the theoretical predictions.

I. Introduction

Since the pioneering work of Howells[1], Applebaum[2] and Widrow[3], there has been considerable activity in the development of adaptive antenna arrays for radar, sonar, communication, spectral estimation, etc.

Though the details differ in the different applications, the main assumptions and processing algorithms are essentially the same. In particular, a key assumption in all the previously cited work is that the interfering signals are not coherent with (i.e., do not have fixed phase differences from) the desired signal. More generally, two signal will be said to be coherent if one is a scaled and delayed replica of the other. Coherent interference can arise when multipath propagation is present, or when "smart" jammers deliberately induce coherent interference, e.g. by retrodirecting the signal energy to the receiver.

Coherence can completely destroy the performance of adaptive array systems. We shall show this by computer simulations in Section IV.

In the reference[4] Evans, Johnson and Sun show that the subaperture sampling or spatial smoothing idea, as they call it, can be applied to the off-line eigenstructure based method (of Bienvenu and of Schmidt) for direction finding. This is an important contribution, the main idea of which we independently rediscovered later (see Shan, Wax and Kailath[5]). However, adaptive versions of this spatial smoothing scheme were not obvious. In this paper we will introduce an on-line spatial smoothing algorithm for the problem of adaptive antenna array system.

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II. The Optimum Weight Vector and Beam Patterns of Beamforming Array for Noncoherent and Coherent Incident Signals

It is easy to show that the optimum weight vector for different criteria is of the form[6]

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{xx}^{-1}\mathbf{a}(\tau_0)$$

where $\mathbf{R}_{xx} = E\mathbf{x}(t)\mathbf{x}^*(t)$ is the covariance matrix of the array measurement vector $\mathbf{x}$, α is a scalar constant, and $\mathbf{a}(\tau_0)$ is so-called "look" direction vector.

The beam pattern of a array of M sensors is obtained by plotting

$$\mathbf{w}_{opt}^*\mathbf{a}(\tau)$$

where

$$\mathbf{a}(\tau) = [1 \quad e^{-j\omega\tau} \; \ldots \; e^{-j(M-1)\omega\tau}]^T$$

$$\tau = d\cos\vartheta/c\,, \quad 0 \le \vartheta \le 2\pi\,.$$

Explicit analytical expressions are difficult to obtain, but we can obtain insight into the interference rejection properties of the array by the following asymptotic (high SNR) analysis.

We shall assume that we are interested in the signal $s(t)$ in the known look direction ϑ_0, and that this signal is statistically independent of the (interfering) signals from the other K-1 unknown directions $\{\vartheta_1,\ldots,\vartheta_{K-1}\}$.

If in addition there are no fixed relations between the phases of signals, or more generally if none of the signals is a scaled and shifted version of any other signals we shall say that the signals are *completely noncoherent*.

Noncoherent Signals

Under the assumptions of statistical independence and of noncoherence, we shall express the array measurement data vector as

$$\mathbf{x}(t) = \mathbf{a}(\tau_0)s(t) + \bar{A}\mathbf{j}(t) + \mathbf{v}(t)$$

where $\bar{A}$ specifies the interference directions

$$\mathbf{X} = [\mathbf{a}(\tau_1) \cdots \mathbf{a}(\tau_{K-1})]$$

and $\mathbf{s}(t)$, the array input signal vector, is consisting of desired signal $s(t)$ and jamming signals $\mathbf{j}(t)$

$$\mathbf{s}(t) = \begin{bmatrix} s(t) \\ \\ \mathbf{j}(t) \end{bmatrix} = \begin{bmatrix} s(t) \\ j_1(t) \\ \vdots \\ j_{K-1}(t) \end{bmatrix} .$$

The covariance of $\mathbf{x}(t)$ can now be written

$$\mathbf{R}_{xx} = p_0^2 \mathbf{a}(\tau_0)\mathbf{a}^*(\tau_0) + \mathbf{R}_{nn}$$

where

$$\mathbf{R}_{nn} = \mathbf{X}\mathbf{R}_{jj}\mathbf{X}^* + \sigma^2 \mathbf{I}$$

and we have assumed, for simplicity, the noise intensity, σ^2, to be same at each sensor.

By using the matrix inversion lemma, we can write

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{xx}^{-1}\mathbf{a}(\tau_0)$$

$$= \alpha \left[\mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0) - p_0^2 \frac{\mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0) \cdot \mathbf{a}^*(\tau_0)\mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0)}{1 + p_0^2 \mathbf{a}^*(\tau_0)\mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0)} \right]$$

$$= \beta \cdot \mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0)$$

where β is a scalar constant.

We also introduce the modal representation

$$\mathbf{X}\mathbf{R}_{jj}\mathbf{X}^* = \sum_{i=1}^{K-1} \lambda_i \mathbf{e}_i \mathbf{e}_i^* ,$$

where $\{\lambda_i\}$ and $\{\mathbf{e}_i\}$ are the nonzero eigenvalues and the corresponding eigenvectors of the $M \times M$ matrix $\mathbf{X}\mathbf{R}_{jj}\mathbf{X}^*$, which will have rank $K-1$ because $\mathbf{R}_{jj}$ is the covariance matrix of the $K-1$ *noncoherent* signals.

Finally we shall also assume that the background measurement noise intensity is small compared to the signals $j(t)$, so that we shall have

$$\lambda_i \gg \sigma^2$$

and

$$\frac{1}{\sigma^2} \gg \frac{1}{\lambda_i + \sigma^2} .$$

Then we can write

$$\mathbf{w}_{opt} = \beta \mathbf{R}_{nn}^{-1}\mathbf{a}(\tau_0)$$

$$= \beta \left\{ \sum_{i=1}^{K-1} \frac{1}{\lambda_i + \sigma^2} \mathbf{e}_i \mathbf{e}_i^* + \sum_{i=K}^{M} \frac{1}{\sigma^2} \mathbf{e}_i \mathbf{e}_i^* \right\} \mathbf{a}(\tau_0)$$

$$\doteq \frac{\beta}{\sigma^2} \sum_{i=K}^{M} \rho_i \mathbf{e}_i \ , \ \ \rho_i = \mathbf{e}_i^* \mathbf{a}(\tau_0) .$$

where $\doteq$ denotes asymptotic (as $\sigma^2 \to 0$) equality. Now by construction, the direction vectors $\{\mathbf{a}(\tau_1), ..., \mathbf{a}(\tau_{K-1})\}$ of the interfering signals, which are the columns of the matrix $\mathbf{X}$, lie in the span of the first $K-1$ eigenvectors $\{\mathbf{e}_1, \ldots, \mathbf{e}_{K-1}\}$ and are therefore **orthogonal** *to the remaining eigenvectors* $\{\mathbf{e}_K, \ldots, \mathbf{e}_M\}$.

Therefore, we shall have

$$\mathbf{w}_{opt}^* \mathbf{a}(\tau_l) \doteq \frac{\beta}{\sigma^2} \sum_{i=K}^{M} \rho_i \mathbf{e}_i^* \mathbf{a}(\tau_l) = 0 \quad l = 1, ..., K-1 ,$$

so that the beam pattern will have "deep nulls" in the interference directions. In the look direction, on the other hand, the constraint will ensure that we have

$$\mathbf{w}_{opt}^* \mathbf{a}(\tau_0) = 1 .$$

This is the well known behavior of the linear array, which can be approximately achieved by a variety of adaptive algorithms[6].

However, the situation deteriorates badly in the coherent case.

Coherent Signals

If the signals have *fixed* phase differences, which rally means equal frequencies and fixed φ_i, then we shall have the representation

$$As = \mathbf{a}(\tau_0)s(t) + \sum_{i=1}^{K-1} \mathbf{a}(\tau_i)j_i(t)$$

$$= [\mathbf{a}(\tau_0) + \gamma_1 \mathbf{a}(\tau_1) + \cdots + \gamma_{K-1}\mathbf{a}(\tau_{K-1})] s(t)$$

where the $\{\gamma_i\}$ are fixed complex constants given by

$$\gamma_i = (p_i / p_0)\, e^{j(\varphi_i - \varphi_0)} , \quad i = 1, ..., K-1$$

In this case, the covariance matrix, $\mathbf{A}E\mathbf{s}\mathbf{s}^*\mathbf{A}^*$, will have rank 1, so that it will have only one nonzero eigenvalue λ_1, and the covariance matrix R_{xx} will have $M-1$ eigenvalues equal to σ^2.

Therefore we shall have

$$\mathbf{w}_{opt} = \alpha \mathbf{R}_{xx}^{-1}\mathbf{a}(\tau_0)$$

$$= \alpha \left[\frac{1}{\lambda_1 + \sigma^2} \mathbf{e}_1 \mathbf{e}_1^* + \sum_{i=2}^{M} \frac{1}{\sigma^2} \mathbf{e}_i \mathbf{e}_i^* \right] \mathbf{a}(\tau_0)$$

$$\doteq \alpha \sum_{i=2}^{M} \frac{1}{\sigma^2} (\mathbf{e}_i^* \mathbf{a}(\tau_0)) \mathbf{e}_i$$

All we can say here is that the linear combination

$$\mathbf{b} := \mathbf{a}(\tau_0) + \gamma_2\mathbf{a}(\tau_1) + \cdots + \gamma_K\mathbf{a}(\tau_{K-1})$$

will be orthogonal to the $\{\mathbf{e}_2, \ldots, \mathbf{e}_M\}$. This does not, however, imply that the same will be true of the $\{\mathbf{a}(\tau_1),\ldots,\mathbf{a}(\tau_{K-1})\}$ individually, and therefore there will not in general be any nulls in the directions of the interfering signals.

Due to the constraint, we shall have

$$\mathbf{w}_{opt}^*\mathbf{a}(\tau_0) = 1 \quad ,$$

but this is of small comfort, because the actual array output will be

$$y(t) = \mathbf{w}_{opt}^*\mathbf{x}(t)$$

$$\doteq \alpha\left[\sum_{i=2}^{M} \frac{1}{\sigma^2}(\mathbf{e}_i^*\mathbf{a}(\tau_i))\mathbf{e}_i^*\right]\left[\mathbf{b}s(t) + v(t)\right]$$

where we recall that $\mathbf{b} := \mathbf{a}(\tau_0) + \sum_{i=2}^{M} \gamma_i\mathbf{a}(\tau_i)$ lies along $\mathbf{e}_1$ and is orthogonal to $\{\mathbf{e}_2, \ldots, \mathbf{e}_M\}$. Therefore there will be no signal output from the conventional array when the signals are coherent.

A Way Out

This analysis also makes clear what is necessary to rescue the situation: we must somehow restore the rank of the covariance matrix $E(\mathbf{A}s)(\mathbf{A}s)^*$ to being K. Then the noise-alone eigenvectors will be orthogonal to all the vectors in the space of the signals (desired signal and interfering signals) and the beam pattern will have nulls in the directions of the interfering signals. A simple scheme for achieving this rank restoration with an adaptive algorithm is proposed in the next section.

III. A New Adaptive Antenna Array System

We shall describe a preprocessing scheme for the sensor outputs that will restore the rank of the signal covariance matrix to K even if the signals are completely coherent with each other.

The scheme is based on combining measurements from overlapping subarrays.

Given the M sensor outputs at any time instant,

$$\mathbf{x}(t) = [x_1(t) \cdots x_M(t)]^T \quad ,$$

define p subsets (recall that K is the number of sources)

$$\mathbf{z}^{(1)}(t) = [x_1(t) \cdots x_{K+1}(t)]^T$$

$$\mathbf{z}^{(2)}(t) = [x_2(t) \cdots x_{K+2}(t)]^T$$

$$\vdots$$

$$\mathbf{z}^{(p)}(t) = [x_p \cdots x_{K+p}(t)]^T \quad .$$

Define

$$\mathbf{R}_{zz}^{(k)} = E\mathbf{z}^{(k)}\mathbf{z}^{(k)*}$$

and the spatial smoothed correlation matrix:

$$\overline{\mathbf{R}} = \frac{1}{p}\sum_{k=1}^{p}\mathbf{R}_{zz}^{(k)}$$

Then we can prove (see Reference[7,5]) that $\overline{\mathbf{R}}$ will have the form

$$\overline{\mathbf{R}} = \mathbf{A}\overline{\mathbf{S}}\mathbf{A}^* + \sigma^2 I$$

where

$\overline{\mathbf{S}}$ will have rank K if and only if $p \geq K$.

Once $\overline{\mathbf{S}}$ has rank K, then the noise eigenvectors will be orthogonal to the columns of $\mathbf{A}$ and by the analysis of Section III, will give nulls in the interference directions. The definition of $\mathbf{z}^{(k)}$ shows that $K + p = M$ and this combines with the constraint on p to require that

$$M = K + p \geq 2K$$

Therefore for this scheme to work, we must have at least twice as many sensors as signal sources in this case.

Coherent Subgroups

If there are some coherent source inputs and some noncoherent inputs, we should divide the sources into G noncoherent subgroups within each of which the inputs are completely coherent. Then we form a matrix

$$\overline{\mathbf{R}} = \overline{\mathbf{R}}_1 + \cdots + \overline{\mathbf{R}}_G$$

To destroy the coherency in all of the groups, the total number of subgroups must be at least equal to the size of the largest subgroup (see reference[7]).

With these results in hand, we can now explain how to do adaptive processing of the sensor outputs.

Adaptive Processing

We can rewrite the expression for the estimate of $\overline{\mathbf{R}}$ from N data snapshots as

$$\mathbf{R}_N = \frac{1}{p}\sum_{k=1}^{p}\left[\frac{1}{N}\sum_{j=1}^{N}\mathbf{z}_j^{(k)}\mathbf{z}_j^{(k)*}\right]$$

$$= \frac{1}{Np}\sum_{j=1}^{N}\sum_{k=1}^{p}\mathbf{z}_j^{(k)}\mathbf{z}_j^{(k)*}$$

where the subscript j denotes the j-th time

instant, i.e.

$$z_j^{(k)} = [\, x_k(j), \ldots\ldots, x_{K-1+k}(j) \,]$$

and the 'hat' signifies an estimated quantity.

Then

$$\hat{R}_{N+1} = \frac{1}{(N+1)p}\sum_{j=1}^{N+1}\sum_{k=1}^{p} z_j^{(k)} z_j^{(k)*}$$

$$= \frac{1}{(N+1)p}\sum_{j=1}^{N}\sum_{k=1}^{p} z_j^{(k)} z_j^{(k)*} + \frac{1}{(N+1)p}\sum_{k=1}^{p} z_{N+1}^{(k)} z_{N+1}^{(k)*}$$

$$= \frac{N}{(N+1)}\hat{R}_N + \frac{1}{(N+1)p}\sum_{k=1}^{p} z_{N+1}^{(k)} z_{N+1}^{(k)*}$$

This expression suggests that we can recursively update the inverse of $\hat{R}$ by using the matrix inversion lemma iteratively p times once for each $\{z^{(k)}\}$. It follows that we can also use approximate gradient-type adaptive algorithms (e.g. the LMS algorithm of Widrow and Hoff) to update the weights of the adaptive processor.

Fig.1a shows how to form the spatial data subset from one 'snapshot'. A flow diagram of the procedure is shown in Fig.1b. At each time instant, the 'snapshot' of M data samples is divided into overlapping subgroups of K samples each; these subgroups are then fed in succession into the adaptive processor, which updates a K-dimensional (if each sensor is followed by a N tapped-delay-line, then a $K \times N$ -dimensional) weight vector each time. After all the subgroups have been processed, the same procedure is repeated with the next data 'snapshot'.

It is important to note that the processor can be any kind of adaptive processor using any adaptive algorithm and any array structure, e.g. the Howells-Applebaum or the Frost arrays could be used.

IV. The Computer Simulation Results

To examine the performance of the suggested beamformers in coherent receiving environments, several computer simulations have been carried out, with that confirm the theoretical predictions.

In our first example, we have a signal $0.1\sin 0.2\pi t$ arriving at 90° and four coherent interfering signals, $10\sin 0.2\pi t$, arriving at 25°, 75°, 120° and 160°. Fig.2a shows the beampattern of a conventional Howells-Applebaum array with six sensors. Our new scheme uses 10 sensors with subarrays of size six, adapted with the same Widrow-Hoff LMS algorithm, and gives the beampattern shown in Fig.2b. The input signal waveform (at arrival angle 90°)is shown in Fig.3a, with the output of the conventional array in Fig.3b and that of our new system in Fig.3c - notice the big difference in the scales of these figures.

In the second example, we consider the case of a wide-band signal of unit power level with central frequency 0.25, bandwidth 0.1, and a narrow-band interfering signal with frequency 0.25, at power level 10 arriving at 45°. A conventional Frost array will suffer considerable spectral and waveform distortion (signal cancellation) in this case; again the new adaptive algorithm gives much better performance. The input signal spectrum is shown in Fig.4a; Fig.4b and Fig.4c show the output spectra of the Frost array and of the new scheme.

Several other simulations have shown similar results.

V. Conclusions

Conventional adaptive antenna arrays perform very poorly in coherent receiving environments. If the received signal is coherent with one interference, the signal will be canceled out on the output of the adaptive antenna system, which will therefore totally fail to operate as a receiving unit. The suggested new adaptive beamforming system is able to overcome this degradation of performance in coherent receiving environments, without considerably increasing the complexity of the system structure or the computational burden. The new array structure can be applied in conjunction with any of the adaptive algorithms and structures of current adaptive arrays, and successfully separates the coherent array inputs, as shown by theoretical analysis and simulation results.

References

1. P.Howells, *Intermediate frequency side-lobe canceller*, U.S.Patent 3202 990 Aug. 24, 1965.
2. S.P.Applebaum, "Adaptive arrays," SPL TR 66-1, Syracuse Univ. Res. Corp. Aug. 1966.
3. B. Widrow, P.E. Mantey, L.J.Griffiths, and B.B.Goode, "Adaptive Antenna systems," *Proc. IEEE* 55 pp. 2143-2159 Dec. 1967.
4. J.E.Evans, J.R.Johnson, and D.F.Sun, "High resolution angular spectrum estimation techniques for terrain scattering analysis and angle of arrival estimation," *Proc. of the First ASSP Workshop on spectral Estimation*, Communication Reseasch Lab., McMaster University, Aug, 1981.
5. T.J.Shan, M.Wax, and T.Kailath, "Eigenstructure Methods For Direction-of-Arrival Estimation of Coherent Sources," *submitted to IEEE Trans. on ASSP*, 1983.
6. R.A.Monzingo and T.W.Miller, *Introduction to adaptive arrays*, John Wiley and Sons, New York 1980.
7. T.J.Shan and T.Kailath, "Adaptive beamforming for coherent signals and interference," *submitted to IEEE Trans on ASSP*, 1983.

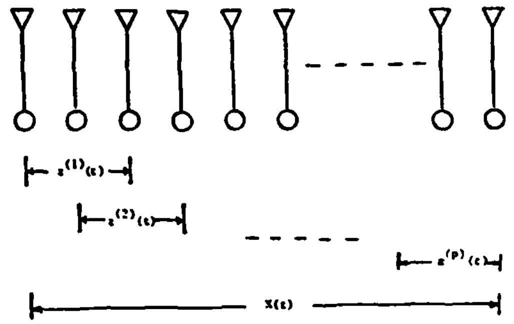

Figure 1a: Subaperture sampling.

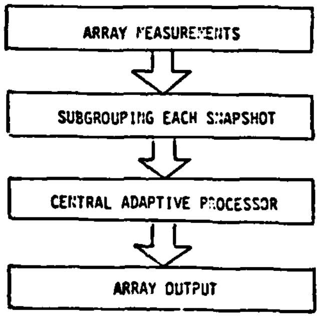

Figure 1b: Flow diagram of the new scheme.

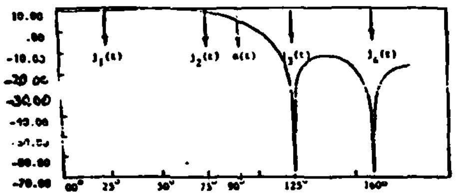

Figure 2a: Beampattern of the Howells-Applebaum array with coherent inputs.

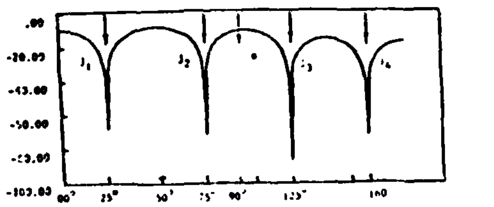

Figure 2b: Beampattern of the new array with the same inputs.

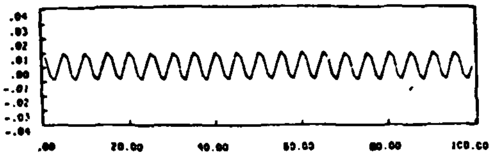

Figure 3a: Input signal.

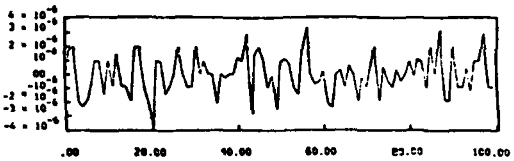

Figure 3b: Outputs signals of the Howells-Applebaum Array.

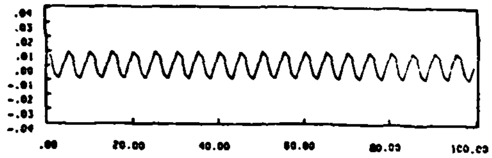

Figure 3c: Output signal of the new array.

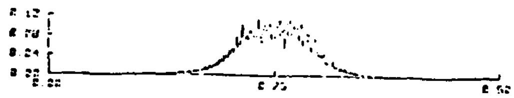

Figure 4a: Input signal spectrum.

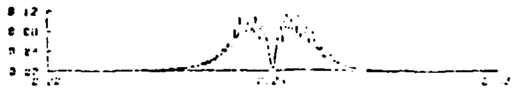

Figure 4b: Frost array output spectrum.

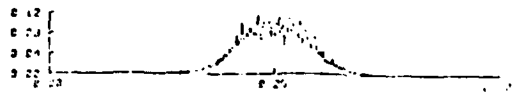

Figure 4c: New array output spectrum.

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